

Name: \_\_\_\_\_  
A2CC: Fractional Exponents

Date: \_\_\_\_\_  
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If  $x \geq 0$  and  $n$  is a positive integer then:

$$(1) \quad x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$(2) \quad x^{\frac{m}{n}} = (\sqrt[n]{x})^m = \sqrt[n]{x^m}$$

$$(3) \quad x^{-\frac{m}{n}} = \frac{1}{(\sqrt[n]{x})^m} = \frac{1}{\sqrt[n]{x^m}}, x \neq 0$$

Examples:

$$1. \quad 225^{\frac{1}{2}}$$

$$2. \quad 27^{\frac{1}{3}}$$

$$3. \quad \left(\frac{64}{27}\right)^{\frac{1}{3}}$$

$$4. \quad 27^{\frac{2}{3}}$$

$$5. \quad 32^{-\frac{3}{5}}$$

$$6. \quad 16^{\frac{3}{2}}$$

$$7. \quad \frac{2^0}{9^{-\frac{3}{2}}}$$

$$8. \quad (-25)^{\frac{1}{2}}$$

$$9. \quad -25^{\frac{1}{2}}$$

$$10. \quad \left(\frac{-8a^3}{b^{-6}}\right)^{\frac{2}{3}}$$

Rewrite each of the following using fractional exponents.

$$11. \quad 5\sqrt{x}$$

$$12. \quad \sqrt[3]{5x^2}$$

$$13. \quad \sqrt[3]{(5x)^2}$$

$$14. \quad \sqrt[5]{x^4y^3}$$

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|--|--|---|--|
| 11. $(2 \times 8)^{\frac{1}{4}}$             | 12. $5(81)^{\frac{1}{4}}$  | 13. $-4(1,000)^{\frac{1}{3}}$   | 14. $49^{\frac{3}{2}}$                     |
| 15. $8^{\frac{5}{3}}$                        | 16. $27^{\frac{4}{3}}$   | 17. $10,000^{\frac{3}{4}}$  | 18. $32^{\frac{4}{3}}$                     |
| 19. $9^{-\frac{1}{2}}$                       | 20. $8^{-\frac{1}{3}}$   | 21. $100^{-\frac{3}{2}}$  | 22. $125^{-\frac{2}{3}}$                   |
| 23. $3^{\frac{1}{2}} \times 3^{\frac{3}{2}}$ | 24. $5^{\frac{1}{3}} \times 5^{\frac{2}{3}}$                       | 25. $7^{\frac{3}{4}} \times 7^{\frac{5}{4}}$  | 26. $4 \times 4^{\frac{1}{2}}$             |
| 27. $32 \times 32^{\frac{1}{2}}$             | 28. $2^{\frac{1}{4}} \times 8^{\frac{1}{4}}$                       | 29. $12^{\frac{5}{3}} \div 12^{\frac{2}{3}}$  | 30. $3^{\frac{7}{3}} \div 3^{\frac{1}{3}}$ |
| 31. $4^{\frac{2}{3}} \div 4^{\frac{1}{6}}$   | 32. $125^{\frac{2}{3}} \div 125^{\frac{1}{3}}$                     | 33. $4^0 + 4^{-\frac{1}{2}}$  | 34. $9^{-2} + 9^{\frac{1}{2}}$             |
| 35. $2[(3)^{-2} + (4)^{-2}]^{-\frac{1}{2}}$  | 36. $(2.3 \times 10^{-\frac{1}{3}})(5.2 \times 10^{-\frac{2}{3}})$ | 37. $\frac{(2(3)^2 + \frac{1}{3^{-2}})^{\frac{3}{2}}}{6(2 + \frac{1}{4})^{-\frac{1}{2}}}$ |  |

In 38–57, write each radical expression as a power with positive exponents and express the answer in simplest form. The variables are positive numbers.

- |                                 |                                   |  |   |
|---------------------------------|-----------------------------------|--|---|
| 38. $\sqrt{7}$                  | 39. $\sqrt{6}$                    | 40. $\sqrt[3]{12}$                         | 41. $\sqrt[3]{15}$                              |
| 42. $\sqrt[4]{3}$               | 43. $\sqrt[5]{2^3}$               | 44. $(\sqrt[5]{9})^4$                      | 45. $\frac{1}{(\sqrt{5})^3}$                    |
| 46. $\sqrt{25a}$                | 47. $\sqrt{49x^2}$                | 48. $\sqrt{64a^3b^6}$                      | 49. $\frac{1}{2}\sqrt{18a^6b^2}$                |
| 50. $\sqrt{9a^{-2}b^6}$         | 51. $\sqrt{\frac{3a}{4b}}$        | 52. $\sqrt[3]{27a^3}$                      | 53. $\sqrt[4]{64x^5}$                           |
| 54. $\frac{1}{\sqrt[5]{xyz^5}}$ | 55. $\sqrt{\frac{9a^{-2}}{4b^4}}$ | 56. $\sqrt[10]{\frac{10^{15}x^{20}}{y^5}}$ | 57. $\sqrt[8]{\sqrt[4]{a} \cdot \sqrt[4]{b^7}}$ |

In 58–73, write each power as a radical expression in simplest form. The variables are positive numbers.

- |   |  |  |   |
|---|--|--|---|
| 58. $3^{\frac{1}{2}}$                                 | 59. $5^{\frac{1}{3}}$  | 60. $6^{\frac{1}{4}}$                                  | 61. $9^{\frac{1}{5}}$   |
| 62. $5^{\frac{3}{2}}$                                 | 63. $12^{\frac{5}{3}}$                                       | 64. $6^{\frac{5}{4}}$                                  | 65. $\frac{1}{5^{\frac{1}{2}}}$   |
| 66. $(x^{13})^{\frac{1}{4}}$                          | 67. $(25x^2y)^{\frac{1}{2}}$                                 | 68. $(50ab^4)^{\frac{1}{2}}$                           | 69. $(16a^5b^6)^{\frac{1}{4}}$  |
| 70. $\frac{(x^5y^6)^{\frac{1}{2}}}{z^{-\frac{1}{3}}}$ | 71. $\frac{5^{\frac{1}{2}}a^{\frac{3}{4}}}{4^{\frac{1}{3}}}$ | 72. $\left(\frac{-32x^{10}}{y^4}\right)^{\frac{1}{5}}$ | 73. $\frac{8^{\frac{1}{5}}a^{\frac{5}{6}}b^{\frac{3}{2}}}{(27c^4)^{\frac{1}{6}}}$ |

In 74–82, write each expression as a power with positive exponents in simplest form.

- |  |   |  |
|--|---|--|
| 74. $\left(\frac{2a^{\frac{1}{2}}}{3a^{\frac{1}{6}}}\right)^6$     | 75. $\left(\frac{x^2y}{3x^4b^2}\right)^{\frac{2}{3}}$ | 76. $\left(\frac{4a^4b^6}{25a^{-1}b}\right)^{\frac{1}{2}}$ |
| 77. $\left(\frac{8a^2z^6}{27x^9a^{-4}z^{-1}}\right)^{\frac{1}{3}}$ | 78. $\sqrt{x^2y} \cdot \sqrt{x^4y^3}$                 | 79. $\frac{\sqrt[6]{a^5}}{\sqrt[5]{a^5}}$                  |
| 80. $\frac{\sqrt[3]{11x^5y^4}}{\sqrt{2x^5y^2}}$                    | 81. $\frac{\sqrt[5]{48xy^2}}{\sqrt[3]{6x^2y^4}}$      | 82. $(\sqrt{2xy^2})(\sqrt[4]{16x^2y})$                     |

83. Verify that the laws for power of a product and power of a quotient are true for the following examples. In each example, evaluate the left side using the rules for radicals and the right side using the rules for fractional exponents:

a.  $(\sqrt[3]{27} \cdot \sqrt[3]{3})^2 \stackrel{?}{=} (27^{\frac{1}{3}} \cdot 3^{\frac{1}{3}})^2$       b.  $\left(\frac{\sqrt[3]{3}}{\sqrt[3]{9}}\right)^3 \stackrel{?}{=} \left(\frac{3^{\frac{1}{3}}}{9^{\frac{1}{3}}}\right)^3$